



WORKSHEET

The Normal Distribution Question – Type 1

Question Description: In this question type they ask you what I would consider simpler normal distribution calculations. Three styles in type 1 are seen in key bellow.

Key:

- ☐ - Normal distribution probability calculations
- ☐ - Finding a constant
- ☐ - Finding 'expected/estimate/average value'

QUESTIONS

Q1 w18_p61

- ☐ (a) It is given that $X \sim N(31.4, 3.6)$. Find the probability that a randomly chosen value of X is less than 29.4. [3]

Q2 w18_p63

The weights of apples sold by a store can be modelled by a normal distribution with mean 120 grams and standard deviation 24 grams. Apples weighing less than 90 grams are graded as 'small'; apples weighing more than 140 grams are graded as 'large'; the remainder are graded as 'medium'.

- ☐ (i) Show that the probability that an apple chosen at random is graded as medium is 0.692, correct to 3 significant figures. [4]

Q3 s18_p63

The diameters of apples in an orchard have a normal distribution with mean 5.7 cm and standard deviation 0.8 cm. Apples with diameters between 4.1 cm and 5 cm can be used as toffee apples.

- (i) Find the probability that an apple selected at random can be used as a toffee apple. [3]

Q4 w16_p61

Packets of rice are filled by a machine and have weights which are normally distributed with mean 1.04 kg and standard deviation 0.017 kg.

- (i) Find the probability that a randomly chosen packet weighs less than 1 kg. [3]
- (ii) How many packets of rice, on average, would the machine fill from 1000 kg of rice? [1]

Q5 w15_p62

- (a) A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

- (i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

Q6 s19_p62

The volume of ink in a certain type of ink cartridge has a normal distribution with mean 30 ml and standard deviation 1.5 ml. People in an office use a total of 8 cartridges of this ink per month. Find the expected number of cartridges per month that contain less than 28.9 ml of this ink. [4]

Q7 w16_p61

The random variable X is such that $X \sim N(20, 49)$. Given that $P(X > k) = 0.25$, find the value of k . [3]

Q8 w19_p62

The heights, in metres, of fir trees in a large forest have a normal distribution with mean 40 and standard deviation 8.

(i) Find the probability that a fir tree chosen at random in this forest has a height less than 45 metres. [2]

(ii) Find the probability that a fir tree chosen at random in this forest has a height within 5 metres of the mean. [2]

Q9 s16_p62

The time in minutes taken by Peter to walk to the shop and buy a newspaper is normally distributed with mean 9.5 and standard deviation 1.3.

(i) Find the probability that on a randomly chosen day Peter takes longer than 10.2 minutes. [3]

(ii) On 90% of days he takes longer than t minutes. Find the value of t . [3]

(iii) Calculate an estimate of the number of days in a year (365 days) on which Peter takes less than 8.8 minutes to walk to the shop and buy a newspaper. [3]

Q10 w16_p62

The time taken to cook an egg by people living in a certain town has a normal distribution with mean 4.2 minutes and standard deviation 0.6 minutes.

(i) Find the probability that a person chosen at random takes between 3.5 and 4.5 minutes to cook an egg. [3]

12% of people take more than t minutes to cook an egg.

(ii) Find the value of t . [3]

Q11 w16_p63

The weights of bananas in a fruit shop have a normal distribution with mean 150 grams and standard deviation 50 grams. Three sizes of banana are sold.

Small: under 95 grams
Medium: between 95 grams and 205 grams
Large: over 205 grams

(i) Find the proportion of bananas that are small. [3]

(ii) Find the weight exceeded by 10% of bananas. [3]

The prices of bananas are 10 cents for a small banana, 20 cents for a medium banana and 25 cents for a large banana.

(iii) (a) Show that the probability that a randomly chosen banana costs 20 cents is 0.7286. [1]

(b) Calculate the expected total cost of 100 randomly chosen bananas. [3]

Q12 w17_p62

In Jimpuri the weights, in kilograms, of boys aged 16 years have a normal distribution with mean 61.4 and standard deviation 12.3.

(i) Find the probability that a randomly chosen boy aged 16 years in Jimpuri weighs more than 65 kilograms. [3]

(ii) For boys aged 16 years in Jimpuri, 25% have a weight between 65 kilograms and k kilograms, where k is greater than 65. Find k . [4]

Q13 w17_p63

Josie aims to catch a bus which departs at a fixed time every day. Josie arrives at the bus stop T minutes before the bus departs, where $T \sim N(5.3, 2.1^2)$.

(i) Find the probability that Josie has to wait longer than 6 minutes at the bus stop. [3]

On 5% of days Josie has to wait longer than x minutes at the bus stop.

(ii) Find the value of x . [3]

Q14 w18_p62

(a) The time, X hours, for which students use a games machine in any given day has a normal distribution with mean 3.24 hours and standard deviation 0.96 hours.

(i) On how many days of the year (365 days) would you expect a randomly chosen student to use a games machine for less than 4 hours? [3]

(ii) Find the value of k such that $P(X > k) = 0.2$. [3]

(iii) Find the probability that the number of hours for which a randomly chosen student uses a games machine in a day is within 1.5 standard deviations of the mean. [3]

Q15 s19_p61

The weight of adult female giraffes has a normal distribution with mean 830 kg and standard deviation 120 kg.

(i) There are 430 adult female giraffes in a particular game reserve. Find the number of these adult female giraffes which can be expected to weigh less than 700 kg. [4]

(ii) Given that 90% of adult female giraffes weigh between $(830 - w)$ kg and $(830 + w)$ kg, find the value of w . [3]

Q16 s19_p63

The time taken, in minutes, by a ferry to cross a lake has a normal distribution with mean 85 and standard deviation 6.8.

(i) Find the probability that, on a randomly chosen occasion, the time taken by the ferry to cross the lake is between 79 and 91 minutes. [3]

(ii) Over a long period it is found that 96% of ferry crossings take longer than a certain time t minutes. Find the value of t . [3]

Q17 w19_p61

The shortest time recorded by an athlete in a 400 m race is called their personal best (PB). The PBs of the athletes in a large athletics club are normally distributed with mean 49.2 seconds and standard deviation 2.8 seconds.

- (i) Find the probability that a randomly chosen athlete from this club has a PB between 46 and 53 seconds. [4]
- (ii) It is found that 92% of athletes from this club have PBs of more than t seconds. Find the value of t . [3]

Q18 s15_p62

- (a) Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.
 - (i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run. [4]
 - (ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t . [3]

Q19 w19_p63

The heights of students at the Mainland college are normally distributed with mean 148 cm and standard deviation 8 cm.

- (i) The probability that a Mainland student chosen at random has a height less than h cm is 0.67. Find the value of h . [3]
- 120 Mainland students are chosen at random.
- (ii) Find the number of these students that would be expected to have a height within half a standard deviation of the mean. [4]

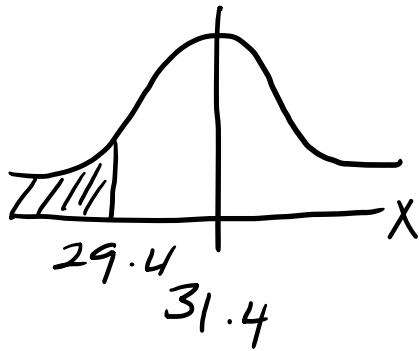
Q20 w17_p61

The weight, in grams, of pineapples is denoted by the random variable X which has a normal distribution with mean 500 and standard deviation 91.5. Pineapples weighing over 570 grams are classified as 'large'. Those weighing under 390 grams are classified as 'small' and the rest are classified as 'medium'.

- (i) Find the proportions of large, small and medium pineapples. [5]
- (ii) Find the weight exceeded by the heaviest 5% of pineapples. [3]
- (iii) Find the value of k such that $P(k < X < 610) = 0.3$. [5]

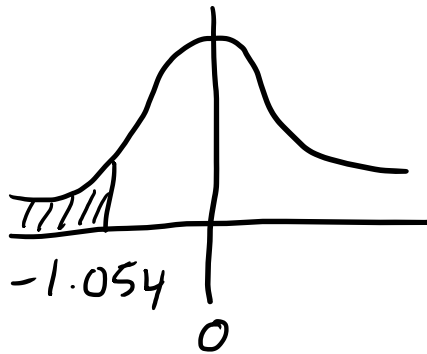
ANSWERS

① $X \sim N(31.4, 3.6)$ $P(X < 29.4) = ?$

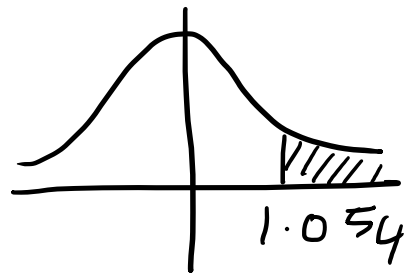


$$\therefore Z = \frac{29.4 - 31.4}{\sqrt{3.6}}$$

$$Z = -1.054$$



=



$$\therefore P(X < 29.4) = P(Z < -1.054)$$

$$\begin{aligned} P(Z < -1.054) &= P(Z > 1.054) \\ &= 1 - P(Z < 1.054) \\ &= 1 - \Phi(1.054) \\ &= 1 - 0.8540 \end{aligned}$$

$$\therefore P(Z < -1.054) = 0.146$$

$$\therefore \boxed{P(X < 29.4) = 0.146}$$

② $X \sim N(120, 24^2)$ $X =$ Weight of randomly selected apple

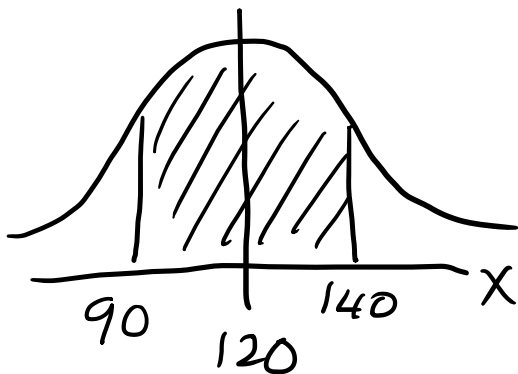
$X < 90$: small

$X > 140$: Large

$\therefore 90 < X < 140$: Medium.

$$\therefore P(\text{Apple is Medium}) = P(90 < X < 140)$$

$$\text{Show : } P(90 < X < 140) = 0.692$$

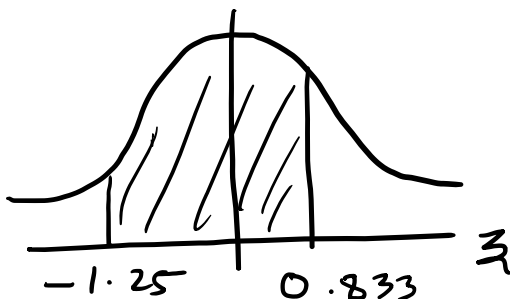


$$\therefore \bar{z}_1 = \frac{90 - 120}{24}$$

$$= -1.25$$

$$\bar{z}_2 = \frac{140 - 120}{24}$$

$$= 0.833$$



$$\therefore P(90 < X < 140) = P(-1.25 < Z < 0.833)$$



$$\therefore P(-1.25 < Z < 0.833) = P(Z < 0.833) - P(Z < -1.25)$$

$$\text{AS } P(Z < -1.25) = 1 - P(Z < 1.25)$$

$$\begin{aligned} P(-1.25 < Z < 0.833) &= P(Z < 0.833) - [1 - P(Z < 1.25)] \\ &= \Phi(0.833) - [1 - \Phi(1.25)] \\ &= 0.7975 - [1 - 0.8944] \\ &= 0.692 \quad (3 \text{ s.f.}) \end{aligned}$$

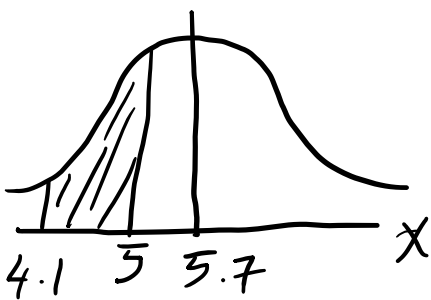
$$\therefore P(90 < X < 140) = 0.692$$

$$\therefore \boxed{P(\text{Apple is Medium}) = 0.692}$$

③ $X \sim N(5.7, 0.8^2)$ $X = \text{Diameter of random Apple}$

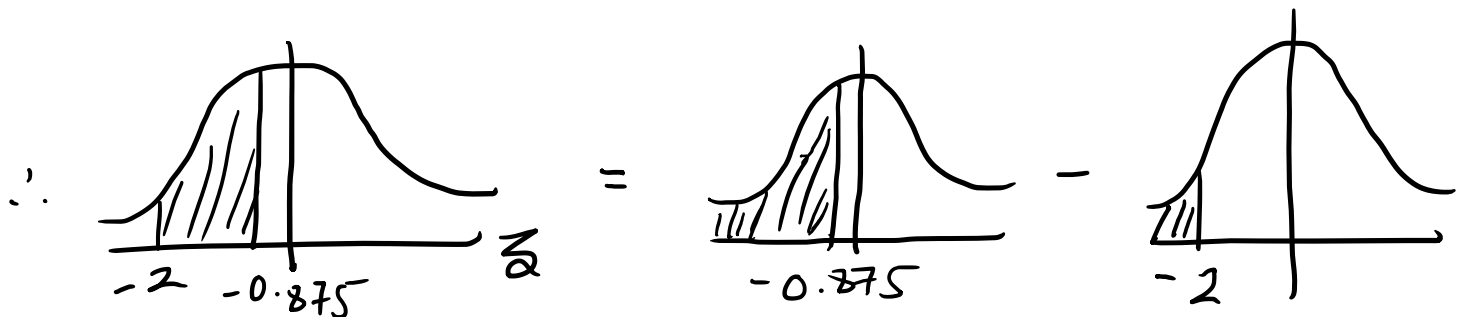
$4.1 < X < 5$: Toffee Apples

$$P(\text{Toffee Apple}) = P(4.1 < X < 5.0)$$



$$Z_1 = \frac{4.1 - 5.7}{0.8} = -2$$

$$Z_2 = \frac{5 - 5.7}{0.8} = -0.875$$



$$P(4.1 < X < 5) = P(-2 < Z < -0.875)$$

$$\therefore P(-2 < Z < -0.875) = P(Z < -0.875) - P(Z < -2)$$

$$\text{As } P(Z < -0.875) = 1 - P(Z < 0.875)$$

$$P(Z < -2) = 1 - P(Z < 2)$$

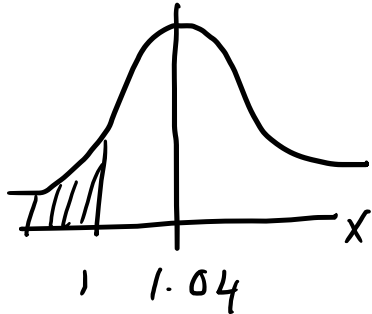
$$\begin{aligned}
 \therefore P(-2 < Z < -0.875) &= [1 - P(Z < 0.875)] - \\
 &\quad [1 - P(Z < 2)] \\
 &= [1 - \Phi(0.875)] - \\
 &\quad [1 - \Phi(2.0)] \\
 &= \Phi(2.0) - \Phi(0.875) \\
 &= 0.9772 - 0.8092 \\
 &= 0.168
 \end{aligned}$$

$$\therefore P(-2 < Z < -0.875) = 0.168$$

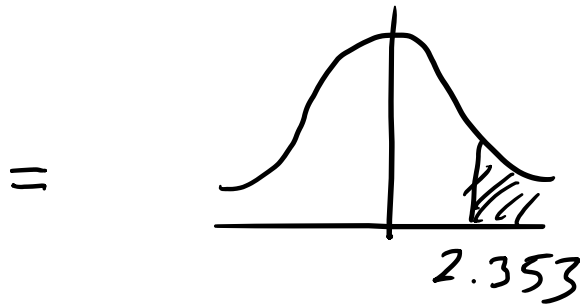
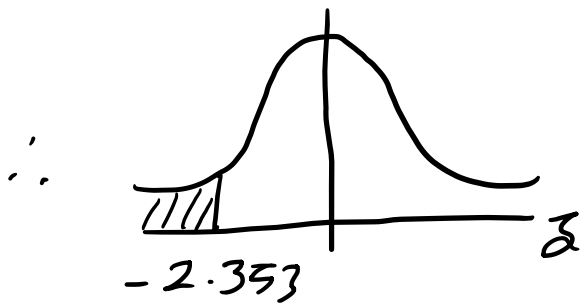
$$\therefore P(4.1 < X < 5) = 0.168$$

$$\therefore \boxed{P(\text{Toffee Apple}) = 0.168}$$

④ $X \sim N(1.04, 0.017^2)$ $X = \text{Weight of random rice packet}$
(i) $P(X < 1) = ?$



$$Z = \frac{1 - 1.04}{0.017} = -2.353$$



$$\begin{aligned}\therefore P(X < 1) &= P(Z < -2.535) \\ &= P(Z > 2.535) \\ &= 1 - P(Z < 2.535) \\ &= 1 - \Phi(2.535) \\ &= 1 - 0.9907 = 0.0093\end{aligned}$$

$$\therefore \boxed{P(X < 1) = 0.0093}$$

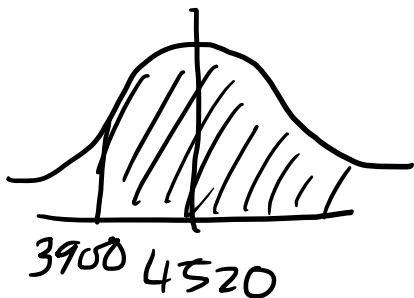
(ii) Average weight of a packet = 1.04 kg

$\therefore$ Average # of packets from 1000 kg = $\frac{1000}{1.04} = 961.5$

Average = 962 packets

(5) $X \sim N(4520, 560^2)$ X = Daily sales of random day.

$P(X > 3900) = ?$



$$Z = \frac{3900 - 4520}{560} = -1.107$$

$\therefore P(X > 3900) = P(Z > -1.107)$

$$\begin{aligned}
 P(Z > -1.107) &= P(Z < 1.107) \\
 &= \Phi(1.107) \\
 &= 0.8657
 \end{aligned}$$

$$\therefore P(X > 3900) = 0.8657$$

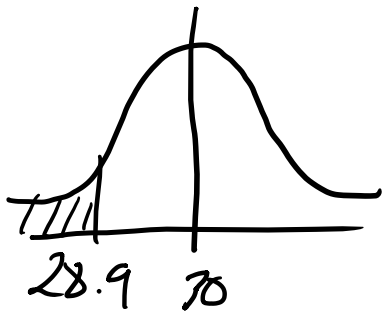
This probability that on a randomly selected day the petrol sales exceed 3900L is "0.8657".

$$\begin{aligned}
 \text{Expected \#} & \\
 \text{of days in a} & = (365)(0.8657) \\
 \text{year} & = 315.9
 \end{aligned}$$

$$\therefore \boxed{\text{Expected} = 316 \text{ days}}$$

⑥ $X \sim N(30, 1.5^2)$ $X = \text{Ink volume in random cartridge}$

$$P(X < 28.9) = ?$$



$$Z = \frac{28.9 - 30}{1.5} = -0.733$$

$$\begin{aligned} \therefore P(X < 28.9) &= P(Z < -0.733) \\ &= P(Z > 0.733) \\ &= 1 - P(Z < 0.733) \\ &= 1 - \Phi(0.733) \\ &= 1 - 0.7682 \\ &= 0.2318 \end{aligned}$$

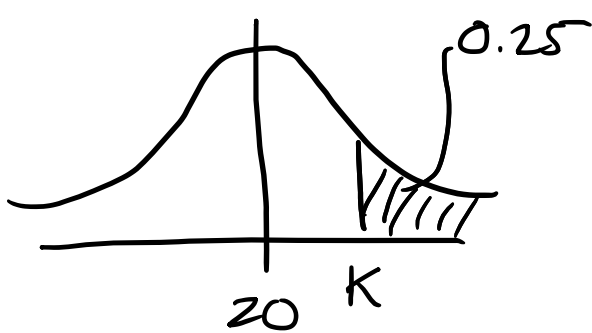
$$\therefore P(X < 28.9) = 0.2318$$

Expected # of cartridges per month that will have less than 28.9ml of ink:

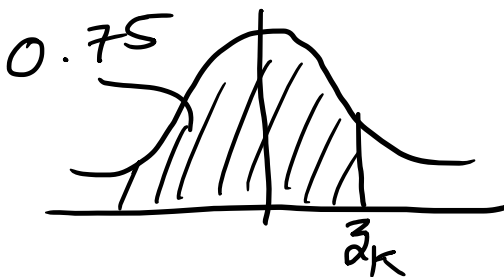
$$\text{Expected} = 8 \times 0.2318 = 1.8544$$

$$\therefore \boxed{\text{Expected} = 2}$$

$$\textcircled{7} \quad X \sim N(20, 49) \quad P(X > k) = 0.25$$



$$z_k = \frac{k - 20}{\sqrt{49}}$$



$$\therefore \Phi(z_k) = 0.75$$

$$\therefore z_k = 0.674$$

$$\therefore 0.674 = \frac{k - 20}{7}$$

$$\boxed{k = 24.7}$$

$$\textcircled{8} X \sim N(40, 8^2)$$

$X = \text{Height of random fir tree}$

$$(i) P(X < 45)$$

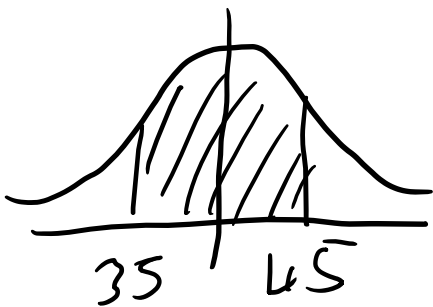


$$Z = \frac{45 - 40}{8} = 0.625$$

$$\begin{aligned} \therefore P(X < 45) &= P(Z < 0.625) \\ &= \Phi(0.625) = 0.734 \end{aligned}$$

$$\therefore \boxed{P(X < 45) = 0.734}$$

$$\begin{aligned} (ii) \quad \mu &= 40 \quad \therefore P(\mu - 5 < X < \mu + 5) \\ &= P(35 < X < 45) \end{aligned}$$



$$Z_1 = 0.625$$

$$Z_2 = -0.625$$

$$\therefore P(35 < X < 45) = P(-0.625 < Z < 0.625)$$

$$P(-0.625 < Z < 0.625) = P(X < 0.625) - P(X < -0.625)$$

$$\text{As } P(X < 0.625) = 0.734$$

$$\text{then } P(X < -0.625) = 1 - 0.734 \\ = 0.266$$

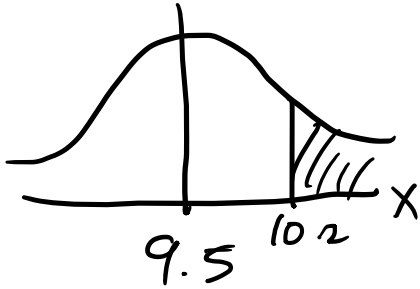
$$\therefore P(35 < X < 45) = 0.734 - 0.266 \\ = 0.468$$

$$\boxed{P(35 < X < 45) = 0.468}$$

$$\textcircled{9} \quad X \sim N(9.5, 1.3^2)$$

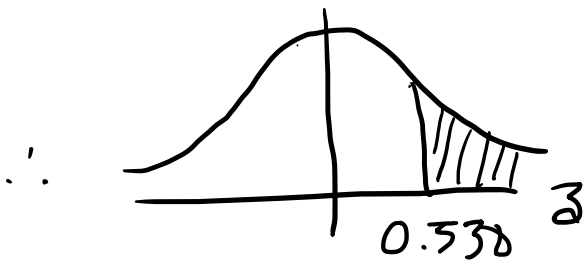
X = Time taken on random day.

$$(i) \quad P(X > 10.2) = ?$$



$$Z = \frac{10.2 - 9.5}{1.3} = 0.538$$

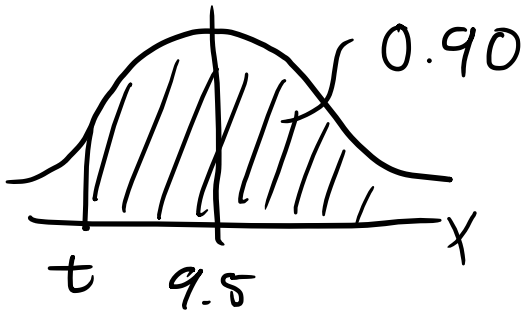
$$P(X > 10.2) = P(Z > 0.538)$$



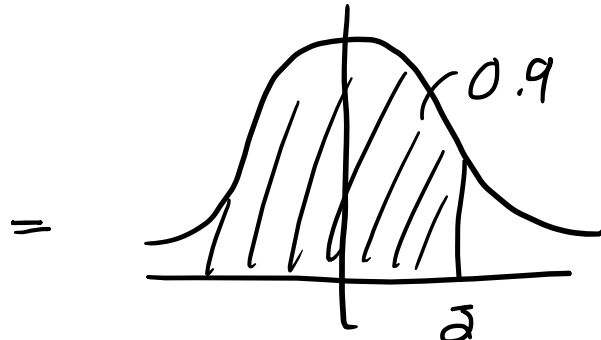
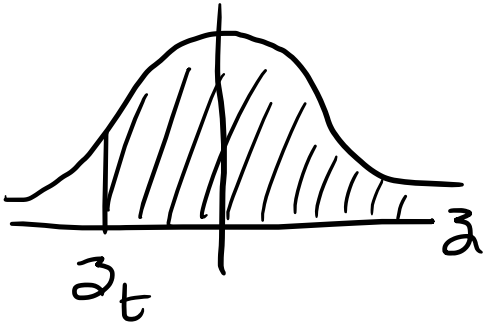
$$\begin{aligned} \therefore P(Z > 0.538) &= 1 - P(Z < 0.538) \\ &= 1 - \Phi(0.538) \\ &= 1 - 0.7046 \\ &= 0.295 \end{aligned}$$

$$\therefore \boxed{P(X > 10.2) = 0.295}$$

$$(ii) \quad P(X > t) = 0.90$$



$$Z_t = \frac{t - 9.5}{1.3}$$



$$\therefore \Phi(Z) = 0.9$$

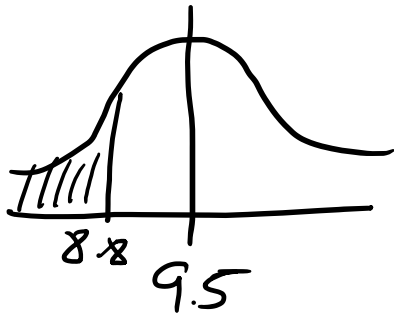
$$Z = 1.282$$

$$\therefore Z_t = -1.282$$

$$\therefore -1.282 = \frac{t - 9.5}{1.3}$$

$$\boxed{t = 7.83}$$

$$(iii) P(X < 8.8) = ?$$



$$Z = \frac{8.8 - 9.5}{1.3} = -0.538$$

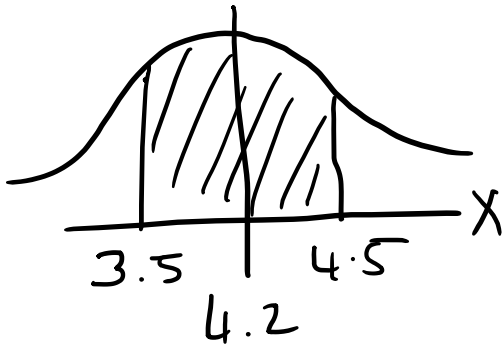
$$\begin{aligned} \therefore P(X < 8.8) &= P(Z < -0.538) \\ &= P(Z > 0.538) \\ &= 0.295 \text{ (from li)} \end{aligned}$$

$$\begin{aligned} \therefore \text{Expected \# of days} &= 0.295 \times 365 \\ &= 107.7 \end{aligned}$$

$$\therefore \boxed{\text{Expected} = 108 \text{ days}}$$

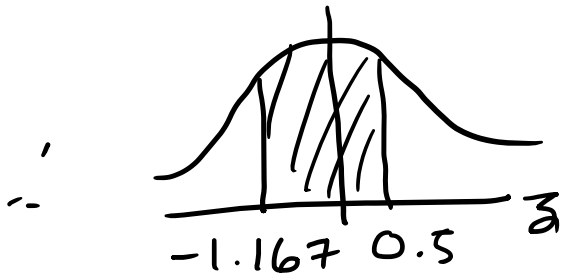
⑩ $X \sim N(4.2, 0.6^2)$ $X = \text{time taken to cook random egg.}$

(i) $P(3.5 < X < 4.5) = ?$



$$z_1 = \frac{3.5 - 4.2}{0.6} = -1.167$$

$$z_2 = \frac{4.5 - 4.2}{0.6} = 0.5$$

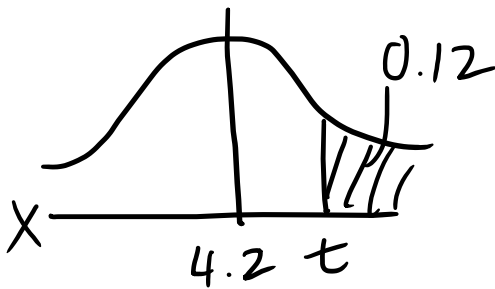


$$P(3.5 < X < 4.5) = P(-1.167 < Z < 0.5)$$

$$\begin{aligned} \therefore P(-1.167 < Z < 0.5) &= P(Z < 0.5) - P(Z < -1.167) \\ &= P(Z < 0.5) - P(Z > 1.167) \\ &= P(Z < 0.5) - [1 - P(Z < 1.167)] \\ &= \Phi(0.5) - [1 - \Phi(1.167)] \\ &= 0.6915 - [0.1206] \\ &= 0.57 \end{aligned}$$

$$\therefore \boxed{P(3.5 < X < 4.5) = 0.57}$$

$$(ii) P(X > t) = 0.12$$



$$z_t = \frac{t - 4.2}{0.6}$$



$$\therefore \Phi(z_t) = 0.88$$

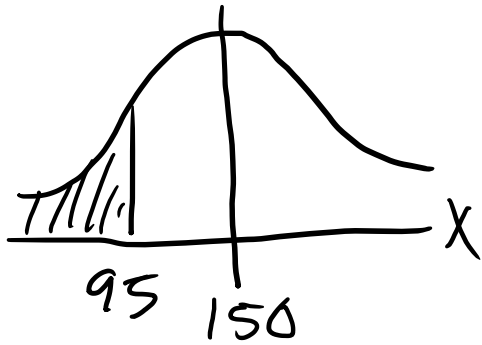
$$\therefore z_t = 1.175$$

$$\therefore 1.175 = \frac{t - 4.2}{0.6}$$

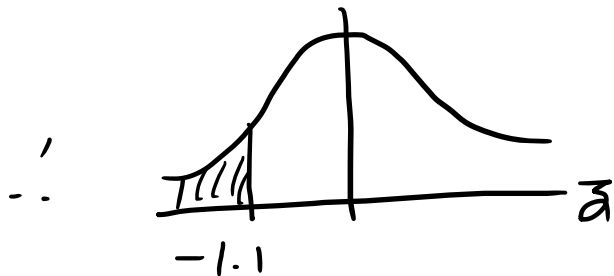
$$\boxed{t = 4.91}$$

① $X \sim N(150, 50^2)$ $X = \text{Weight of random banana}$

(i) $P(\text{small}) = P(X < 95)$



$$Z = \frac{95 - 150}{50} = -1.1$$



$$\therefore P(X < 95) = P(Z < -1.1)$$

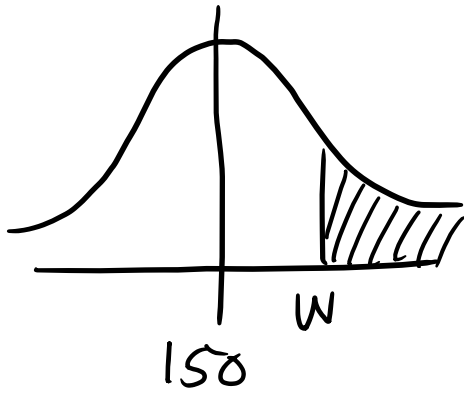
$$\begin{aligned} P(Z < -1.1) &= P(Z > 1.1) \\ &= 1 - P(Z < 1.1) \\ &= 1 - \Phi(1.1) \\ &= 1 - 0.8643 = 0.136 \end{aligned}$$

$$\therefore P(X < 95) = 0.136$$

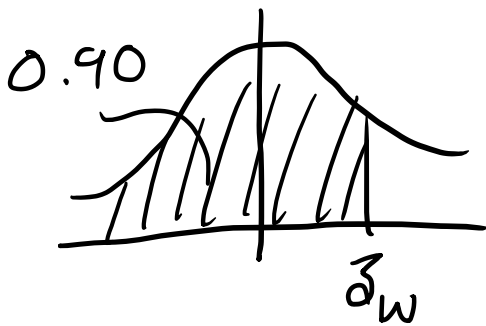
$$\boxed{P(\text{small}) = 0.136}$$

(ii) W = weight exceeded by 10% of bananas

$$P(X > W) = 0.10$$



$$z_w = \frac{W - 150}{50}$$



$$\therefore \Phi(z_w) = 0.90$$

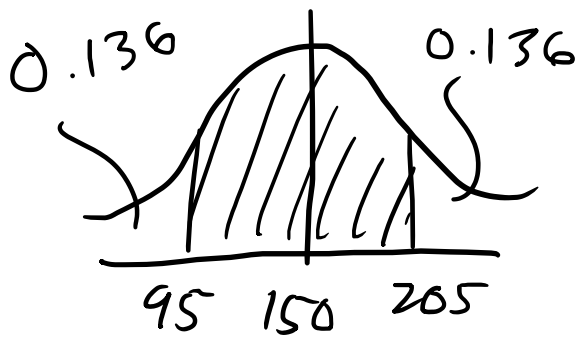
$$\therefore z_w = 1.282$$

$$\therefore 1.282 = \frac{W - 150}{50}$$

$$\boxed{W = 214g}$$

(iii)(a) 20¢: Medium banana

$$P(\text{cost } 20¢) = P(\text{Medium banana}) \\ = P(95 < X < 205)$$



From (i) $P(X < 95) = 0.136$

$$\therefore P(95 < X < 205) = 1 - (2 \times 0.136) \\ \boxed{P(20¢) = 0.728}$$

(b) $P(\text{small}) = 0.136$

$$P(\text{Medium}) = 0.728$$

$$P(\text{Large}) = 1 - 0.136 - 0.728 = 0.136$$

Expected # of small = $0.136 \times 100 = 13.6$

Expected # of Medium = $0.728 \times 100 = 72.8$

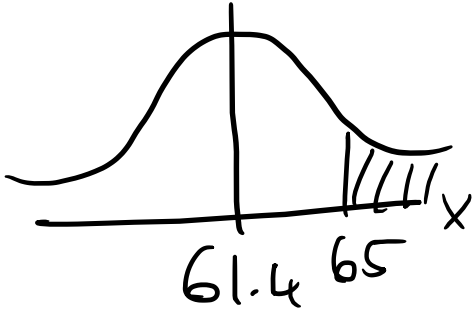
$$\begin{array}{l} \text{Expected \#} \\ \text{of LARUE} \end{array} = 13.6$$

$$\begin{aligned} \therefore \text{Expected cost} &= (13.6)(0.10¢) + (72.8)(0.20¢) \\ &\quad + (13.6)(0.25¢) \\ &= 19.32 \end{aligned}$$

$$\boxed{\text{Expected} = \$19.30¢}$$

⑫ $X \sim N(61.4, 12.3^2)$ $X = \text{Weight of random bag.}$

(i) $P(X > 65) = ?$



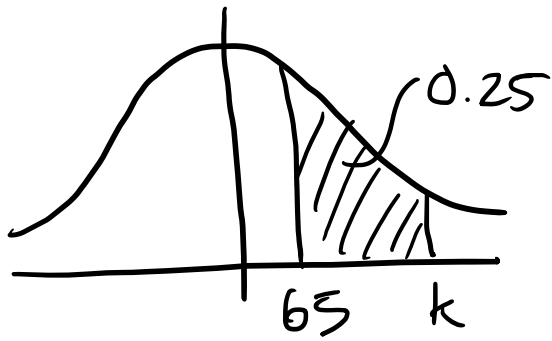
$$z = \frac{65 - 61.4}{12.3} = 0.293$$

$$\therefore P(X > 65) = P(Z > 0.293)$$

$$\begin{aligned} P(Z > 0.293) &= 1 - P(Z < 0.293) \\ &= 1 - \Phi(0.293) \\ &= 1 - 0.6153 \\ &= 0.385 \end{aligned}$$

$$\boxed{P(X > 65) = 0.385}$$

$$(ii) \quad P(65 < X < k) = 0.25$$



$$* P(65 < X < k) = P(X < k) - P(X < 65)$$

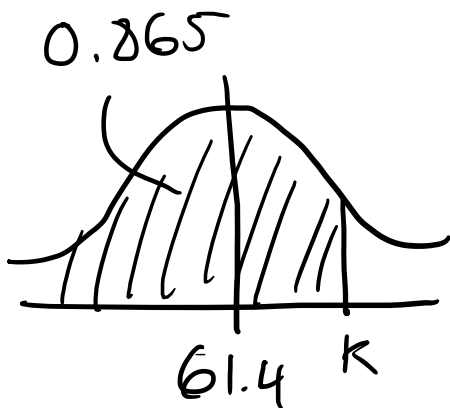
$$P(X < 65) = 1 - P(X > 65) = 1 - 0.385$$

$$\therefore P(X < 65) = 0.615$$

$$\therefore * P(X < k) = P(65 < X < k) + P(X < 65)$$

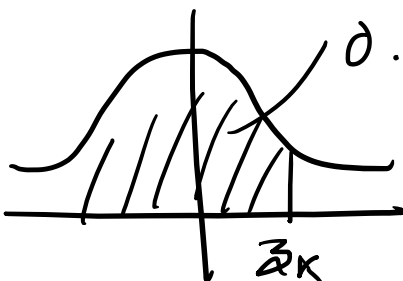
$$= 0.25 + 0.615$$

$$= 0.865$$



$$z_k = \frac{k - 61.4}{12.3}$$

$$\therefore \Phi(z_k) = 0.865$$



$$\therefore z_k = 1.105$$

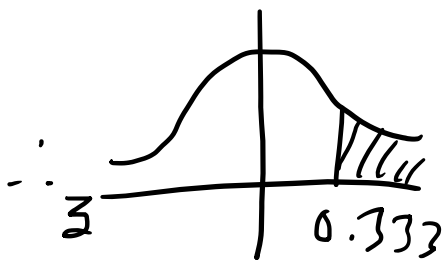
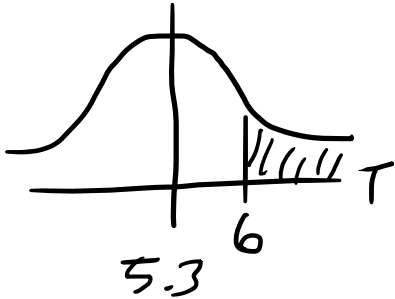
$$\therefore 1.105 = \frac{k - 61.4}{12.3}$$

$$\boxed{k = 75.0}$$

$$(13) T \sim N(5.3, 2.1^2)$$

$$(i) P(T > 6) = ?$$

$$Z = \frac{6 - 5.3}{2.1} = 0.333$$

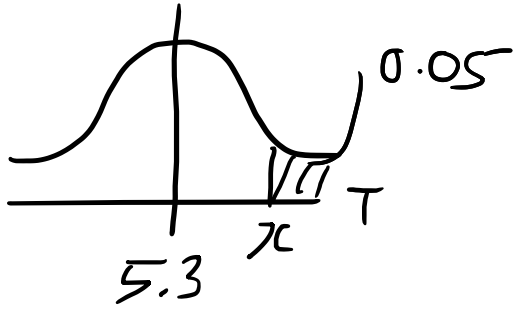


$$\therefore P(T > 6) = P(Z > 0.333)$$

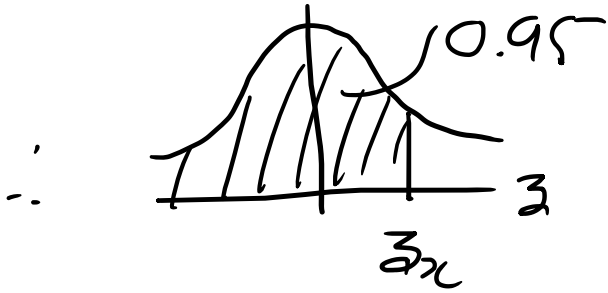
$$\begin{aligned} P(Z > 0.333) &= 1 - P(Z < 0.333) \\ &= 1 - \Phi(0.333) \\ &= 1 - 0.6304 \\ &= 0.370 \end{aligned}$$

$$\therefore \boxed{P(T > 6) = 0.370}$$

$$(ii) P(T > x) = 0.05$$



$$Z_x = \frac{x - 5.3}{2.1}$$



$$\therefore \Phi(Z_x) = 0.95$$

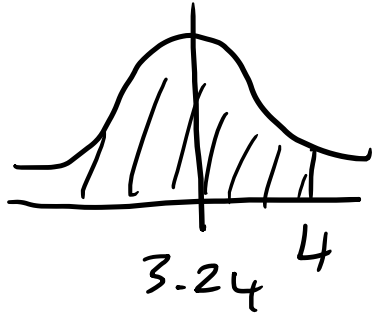
$$\therefore Z_x = 1.645$$

$$\therefore 1.645 = \frac{x - 5.3}{2.1}$$

$$\therefore \boxed{x = 8.75 \text{ Minutes}}$$

①④ $X \sim N(3.24, 0.96^2)$ $X = \text{Hours}$
random student
uses machine

(i) $P(X < 4) = ?$



$$Z = \frac{4 - 3.24}{0.96}$$

$$Z = 0.792$$

$$\therefore P(X < 4) = P(Z < 0.792)$$

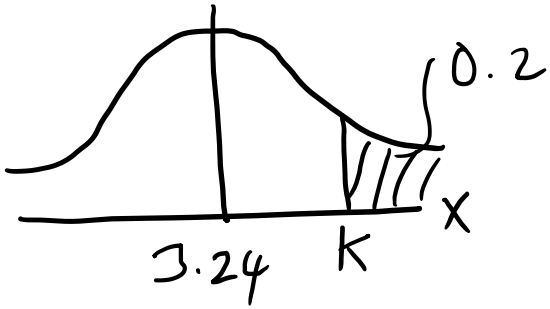
$$P(Z < 0.792) = \Phi(0.792) \\ = 0.7858$$

$$\therefore P(X < 4) = 0.7858$$

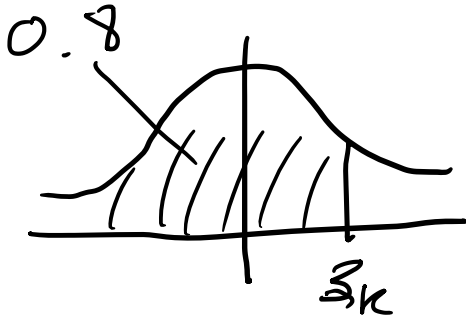
$$\therefore \text{Expected \# of days} = 0.7858 \times 365 \\ = 286.8$$

$\text{Expected} = 287 \text{ days}$

$$(ii) P(X > K) = 0.2$$



$$Z_K = \frac{K - 3.24}{0.96}$$



$$\therefore \Phi(Z_K) = 0.8$$

$$\therefore Z_K = 0.842$$

$$\therefore 0.842 = \frac{K - 3.24}{0.96}$$

$$\boxed{K = 4.05}$$

$$(ii) \sigma = 0.96 \quad \mu = 3.24$$

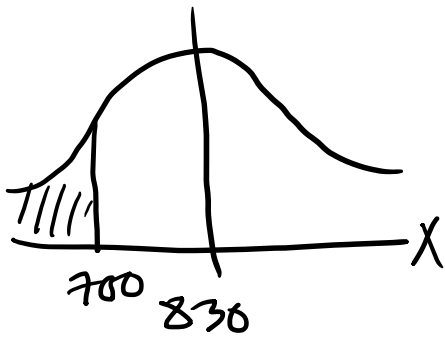
$$P(\mu - 1.5\sigma < X < \mu + 1.5\sigma) = P(-1.5 < Z < 1.5) \\ (\text{without calculation})$$

$$\begin{aligned} \therefore P(-1.5 < Z < 1.5) &= P(Z < 1.5) - P(Z < -1.5) \\ &= P(Z < 1.5) - (1 - P(Z < 1.5)) \\ &= \Phi(1.5) - (1 - \Phi(1.5)) \\ &= 2\Phi(1.5) - 1 \\ &= 2(0.9332) - 1 \\ &= 0.866 \end{aligned}$$

$$\therefore \boxed{P(-1.5 < Z < 1.5) = 0.866}$$

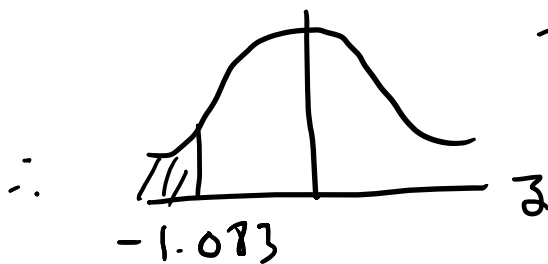
$$(15) X \sim N(830, 120^2)$$

(i) $P(X < 700)$ = Probability a randomly selected giraffe weighs less than 700 kg.



$$Z = \frac{700 - 830}{120} = -1.083$$

$$\therefore P(X < 700) = P(Z < -1.083)$$



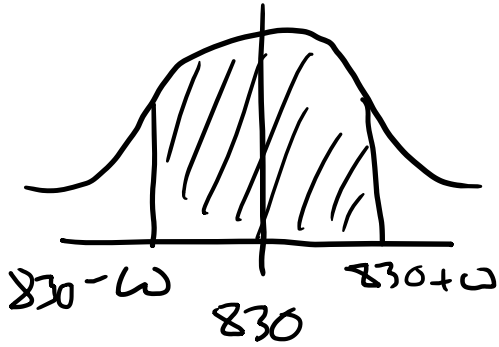
$$\begin{aligned} P(Z < -1.083) &= 1 - P(Z < 1.083) \\ &= 1 - \Phi(1.083) \\ &= 1 - 0.8606 \\ &= 0.1394 \end{aligned}$$

$$P(X < 700) = 0.1394$$

$$\text{Expected \#} = 436 \times 0.1394$$

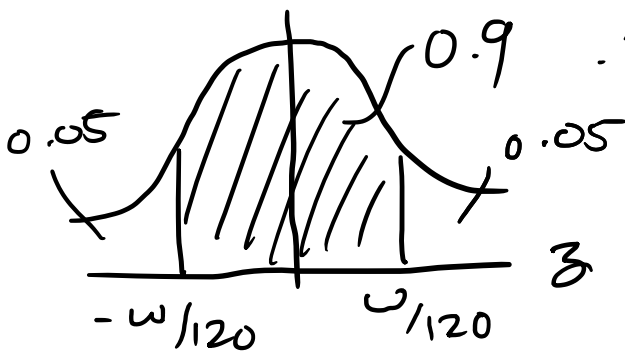
$$\boxed{\text{Expected} = 60}$$

$$(ii) \quad P(830 - w < X < 830 + w) = 0.90$$

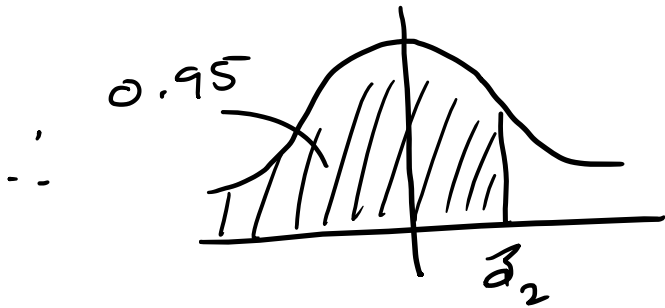


$$Z_1 = \frac{(830 - w) - 830}{120}$$

$$Z_1 = -\frac{w}{120}$$



$$\therefore Z_2 = \frac{w}{120}$$



$$\Phi(Z_2) = 0.95$$

$$Z_2 = 1.645$$

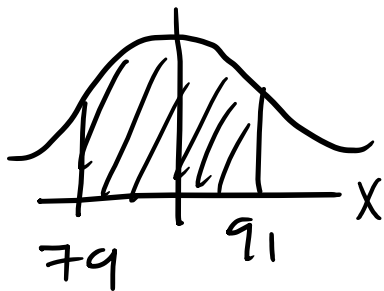
$$\therefore 1.645 = \frac{w}{120}$$

$$w = 197.4$$

$$\boxed{w = 197 \text{ kg}}$$

$$(16) \quad X \sim N(85, 6.8^2)$$

$$(i) \quad P(79 < X < 91) = ?$$



$$z_1 = \frac{79 - 85}{6.8} = -0.882$$

$$z_2 = \frac{91 - 85}{6.8} = 0.882$$

$$\therefore P(79 < X < 91) = P(-0.882 < Z < 0.882)$$

$$P(-0.882 < Z < 0.882) = P(Z < 0.882) - P(Z < -0.882)$$

$$\begin{aligned} \text{As: } P(Z < -0.882) &= P(Z > 0.882) \\ &= 1 - P(Z < 0.882) \end{aligned}$$

$$\therefore P(-0.882 < Z < 0.882) = P(Z < 0.882) - (1 - P(Z < 0.882))$$

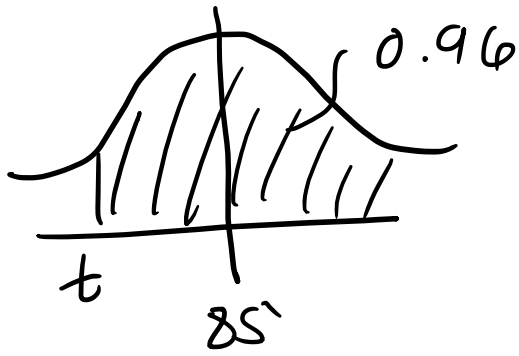
$$= 2P(Z < 0.882) - 1$$

$$= 2\Phi(0.882) - 1$$

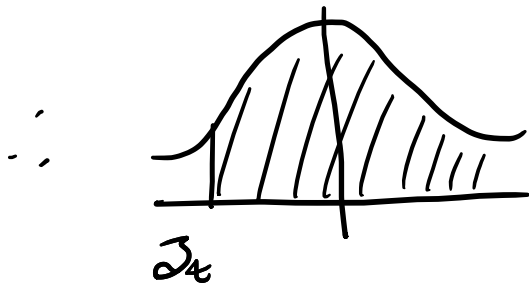
$$= 2(0.8111) - 1$$

$$\therefore \boxed{P(79 < X < 91) = 0.622}$$

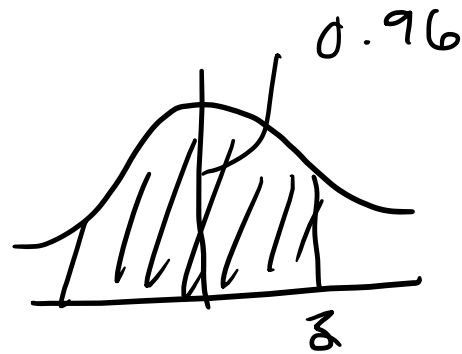
(ii) $P(X > t) = 0.96$



$$Z_t = \frac{t - 85}{6.8}$$



=



$$\therefore \Phi(z) = 0.96$$

$$z = 1.751$$

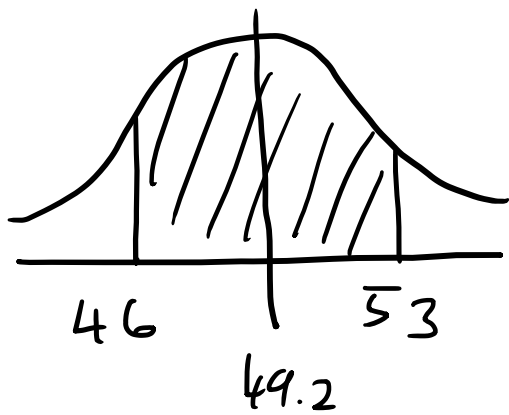
$$\therefore Z_t = -1.751$$

$$\therefore -1.751 = \frac{t - 85}{6.8}$$

$t = 73.1$

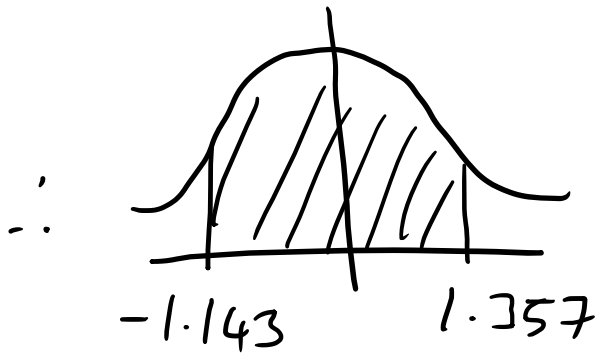
(17) $X \sim N(49.2, 2.8^2)$ $X = \text{PB of a random athlete.}$

(i) $P(46 < X < 53)$



$$z_1 = \frac{46 - 49.2}{2.8} = -1.143$$

$$z_2 = \frac{53 - 49.2}{2.8} = 1.357$$

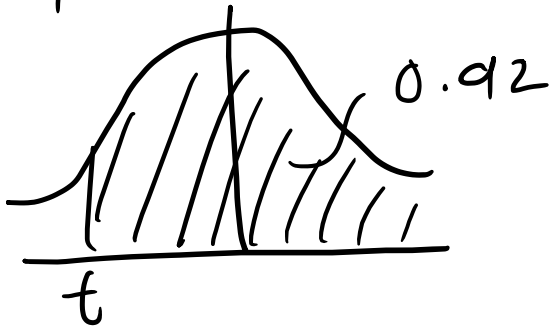


$$P(46 < X < 53) = P(-1.143 < Z < 1.357)$$

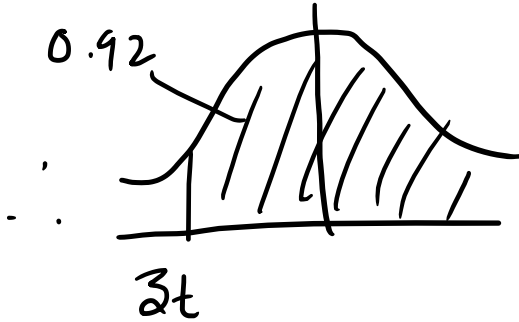
$$\begin{aligned} P(-1.143 < Z < 1.357) &= P(Z < 1.357) - (1 - P(Z < 1.143)) \\ &= \Phi(1.357) - (1 - \Phi(1.143)) \\ &= 0.9126 - (1 - 0.8735) \\ &= 0.786 \end{aligned}$$

$$\therefore \boxed{P(46 < X < 53) = 0.786}$$

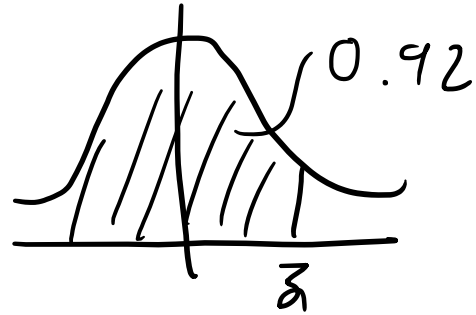
$$(ii) \quad p(X > t) = 0.92$$



$$Z_t = \frac{t - 49.2}{2.8}$$



$\approx$



$$\Phi(Z) = 0.92$$

$$\therefore Z = 1.406$$

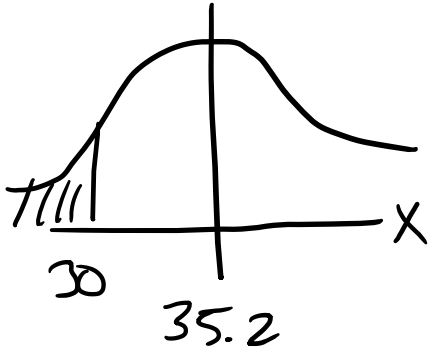
$$\therefore Z_t = -1.406$$

$$\therefore -1.406 = \frac{t - 49.2}{2.8}$$

$$\therefore \boxed{t = 45.3 \text{ seconds}}$$

18 $X \sim N(35.2, 4.7^2)$

(i) $P(X < 30)$

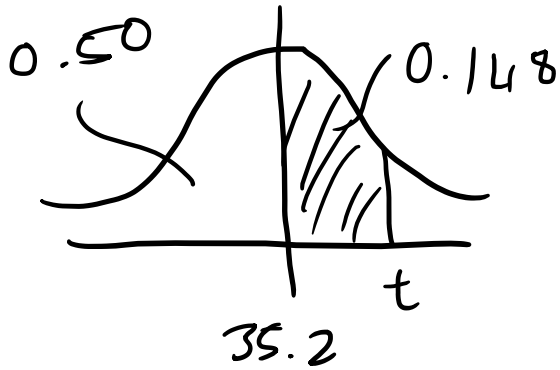


$$Z = \frac{30 - 35.2}{4.7} = -1.106$$

$$\begin{aligned} \therefore P(X < 30) &= P(Z < -1.106) \\ &= 1 - P(Z < 1.106) \\ &= 1 - \Phi(1.106) \\ &= 1 - 0.8655 \\ &= 0.1345 \end{aligned}$$

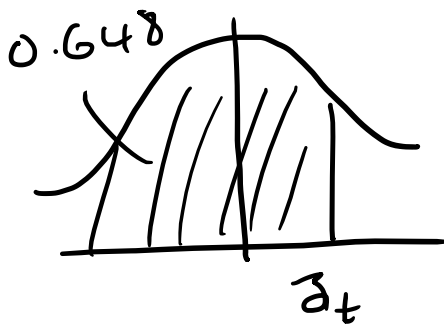
$$\therefore \boxed{P(X < 30) = 0.1345}$$

$$(ii) \quad P(35.2 < X < t) = 0.148$$



$$\begin{aligned} \therefore P(X < t) &= 0.148 + 0.5 \\ &= 0.648 \end{aligned}$$

$$\therefore Z_t = \frac{t - 35.2}{4.7}$$



$$\therefore \Phi(Z_t) = 0.648$$

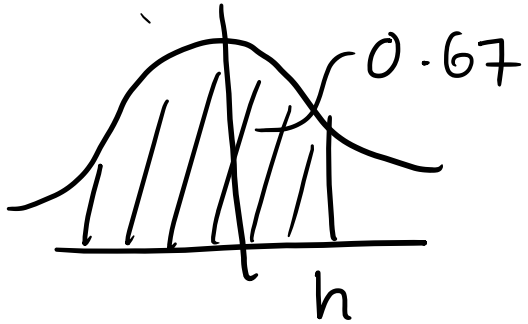
$$Z_t = 0.380$$

$$\therefore 0.380 = \frac{t - 35.2}{4.7}$$

$$\therefore \boxed{t = 37.0}$$

$$(19) \quad X \sim N(148, 8^2)$$

$$(i) \quad P(X < h) = 0.67$$



$$z_h = \frac{h - 148}{8}$$

$$\Phi(z_h) = 0.67 \quad \therefore z_h = 0.44$$

(from table)

$$\therefore 0.44 = \frac{h - 148}{8}$$

$$h = 151.5$$

$$\therefore \boxed{h = 152 \text{ cm}}$$

$$(ii) \quad n = 120 \quad P\left(\mu - \frac{\sigma}{2} < x < \mu + \frac{\sigma}{2}\right)$$

$$\therefore \text{Find } P\left(-\frac{1}{2} < z < \frac{1}{2}\right)$$

$$\begin{aligned} P\left(-\frac{1}{2} < z < \frac{1}{2}\right) &= \Phi(0.5) - (1 - \Phi(0.5)) \\ &= 2\Phi(0.5) - 1 \\ &= 2(0.6915) - 1 \\ &= 0.383 \end{aligned}$$

$$\boxed{P(-0.5 < z < 0.5) = 0.383}$$

(20) $X \sim N(500, 91.5^2)$ $X =$ Weight of random pineapple

Large : $X > 570$

Small : $X < 390$

Medium : $390 < X < 570$

(i) First lets Find $P(X < 570)$

$$Z = \frac{570 - 500}{91.5} = 0.765$$

$$\begin{aligned}\therefore P(X < 570) &= P(Z < 0.765) \\ &= \Phi(0.765) = 0.779\end{aligned}$$

$$\therefore *P(X < 570) = 0.779$$

Also find $P(X < 390)$:

$$Z = \frac{390 - 500}{91.5} = -1.202$$

$$\begin{aligned}\therefore P(X < 390) &= P(Z < -1.202) \\ &= 1 - P(Z < 1.202) \\ &= 1 - \Phi(1.202) \\ &= 1 - 0.8853 =\end{aligned}$$

$$* P(X < 390) = 0.1147$$

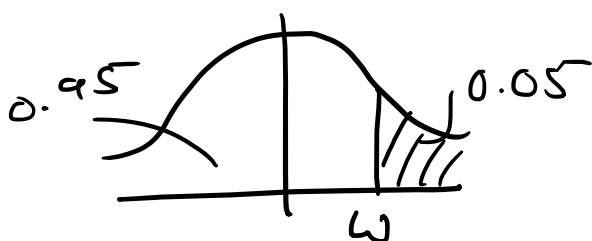
$$\therefore P(\text{small}) = P(X < 390) = 0.1147$$

$$\begin{aligned} P(\text{Large}) &= P(X > 570) = 1 - P(X < 570) \\ &= 1 - 0.779 \\ &= 0.221 \end{aligned}$$

$$\begin{aligned} P(\text{medium}) &= P(390 < X < 570) \\ &= 1 - (0.115 + 0.221) \\ &= 0.663 \end{aligned}$$

$\begin{aligned} P(\text{small}) &= 0.115 \\ P(\text{medium}) &= 0.663 \\ P(\text{Large}) &= 0.221 \end{aligned}$

$$(ii) P(X > w) = 0.05$$



$$Z_w = \frac{w - 500}{91.5}$$

$$\Phi(Z_w) = 0.95$$

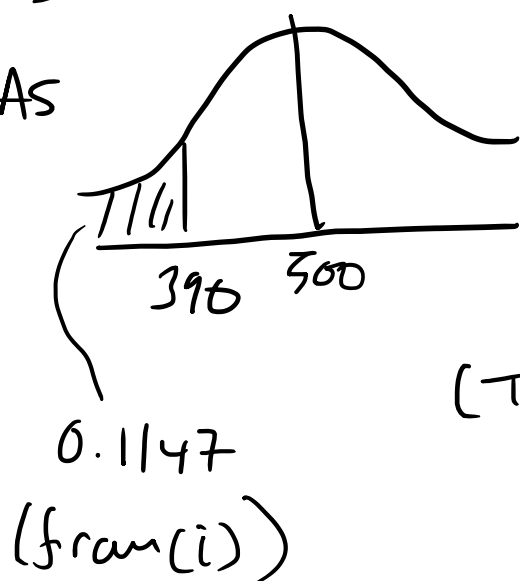
$$\therefore \bar{z}_w = 1.645$$

$$\therefore 1.645 = \frac{w - 500}{91.5}$$

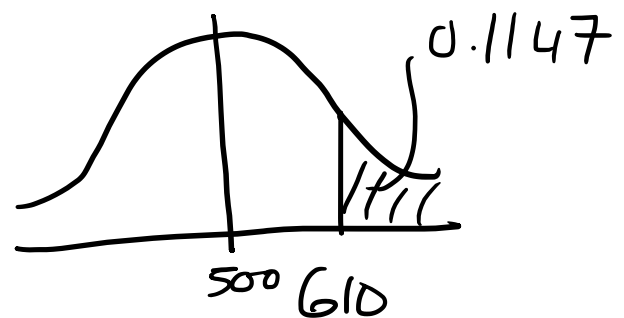
$$\boxed{w = 651 \text{ grams}}$$

$$(iii) P(k < x < 610) = 0.3$$

As



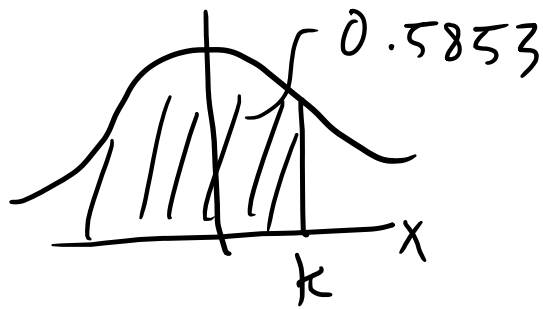
=



(Through symmetry.)

$$\begin{aligned} P(k < x < 610) &= 1 - (P(x > 610) + P(x < k)) \\ &= 1 - (0.1147 + P(x < k)) \\ 0.3 &= 0.8853 - P(x < k) \end{aligned}$$

$$\therefore P(X < k) = 0.5853$$



$$\therefore z_k = \frac{k - 500}{91.5}$$

$$\Phi(z_k) = 0.5853$$

$$\therefore z_k = 0.215$$

$$\therefore 0.215 = \frac{k - 500}{91.5}$$

$$\boxed{k = 519.7}$$